

Andreev scattering and cotunneling between two superconductor-normal metal interfaces: the dirty limit

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Abstract. Crossed Andreev reflections and cotunneling occur between two neighbouring superconductor-normal metal or superconducting-ferromagnet interfaces. Previous works assumed a clean BCS superconductor. Here the calculation of the corresponding crossed conductance terms is generalized to a dirty superconductor. The range of the effect is shown to be the coherence length $\tilde{\xi} = \sqrt{\hbar D/\Delta}$, instead of the BCS coherence length ξ_0 . Moreover, in three dimensions, the algebraic prefactor scales as $1/r$ instead of $1/r^2$. The calculation involves the virtual diffusion probability of quasiparticles below the superconducting gap, in the normal and the anomalous channel.

PACS. 74.45.+c Proximity effects; Andreev effect; SN and SNS junctions – 73.63.Rt Nanoscale contacts – 74.78.-w Superconducting films and low-dimensional structures

Introduction

Hybrid structures involving superconductors and normal or ferromagnetic metals have received considerable interest in the context of spintronics [1]. More recently, multiterminal structures were considered, where two metallic leads are connected at a small distance to the same superconductor [2, 4, 3, 5–9]. Coherent scattering may occur between those leads, leading to original crossed conductance channels [10, 5]. The first one generalizes Andreev reflection: an electron (resp. hole) incident on either contact is reflected as a hole (resp. electron) in the other one. This amounts to having a Cooper pair transferred to (from) the superconductor, each electron of the pair passing at a different contact in the same direction. One may also have an electron (hole) reflected as an electron (hole) from one contact to the other. This process which generalizes normal reflection has been named cotunneling since, in presence of the superconducting gap, a quasiparticle propagates in the superconductor as an evanescent state, in a way similar to what happens in presence of Coulomb blockade [11]. Notice that here cotunneling is essentially elastic. The calculation of the scattering amplitudes and the corresponding non-local conductances was performed in references [5, 8] in the case of a clean BCS superconductor. Normal reflections lead to cotunneling, which conserves spin, while anomalous reflections lead to crossed Andreev conductance involving opposite spin channels in the two leads. These effects give rise to a variety of new phenomena and

potential applications. On one hand, when the leads are spin-polarized, the symmetry between these two processes is broken and interesting nonlocal magnetoconductance effects have been predicted [4, 5]. They can be used as a novel principle for a spin-sensitive STM [9]. On the other hand, crossed Andreev processes, as they lead to spatially separated singlet pairs, have signatures in crossed noise correlations [12–14]. They can also be used as a source of entangled electron pairs, a crucial resource for the treatment of quantum information [15, 16].

In order to maximize the crossed conductance effects, it is essential to optimize the physical regimes, the parameters and the geometry. For instance, for point contacts, the dependence of the crossed conductances with the contact distance r is found to be $\sim (\frac{1}{k_F r})^2 e^{-2r/\pi\xi_0}$ where k_F is the Fermi wavevector and ξ_0 the BCS coherence length [5, 15]. This result is valid for a clean three-dimensional superconductor, and the algebraic prefactor reduces very strongly the amplitude of the effect for realistic distances. For a $2D$ (resp. $1D$) superconductor, the exponent in the prefactor is instead found to be 1 (resp. 0), offering a neat advantage. This led to the proposal of inducing superconductivity in carbon nanotubes, in order to reach an effective $1D$ geometry [17, 18]. Another possibility is to use extended contacts, which cancels the prefactor (see for instance [9]).

In the present work, the calculation of the crossed conductances is generalized to a diffusive superconductor. If the mean-free path l is smaller than the bare coherence length ξ_0 , the question is: which one of the typical lengths, l or the coherence length $\xi = \sqrt{\hbar D/\Delta} \sim \sqrt{\xi_0 l}$, controls

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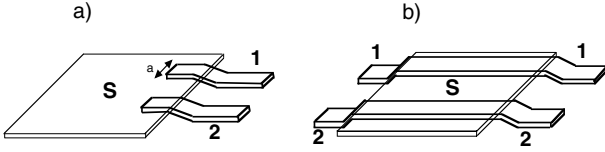


Fig. 1. Schema of a set-up with two neighboring S/N junctions. 1 and 2 denote normal leads; a) point-like contacts; b) extended contacts.

the range of the effect? The calculation involves the diffusion of (evanescent) quasiparticles between the two contacts. As a result, the range is found to be of order ξ . Although not really surprising, this result is not obvious a priori: ξ is known to be the typical length for the variations of the superconducting order parameter, while we are interested here into the damping length for quasiparticles. In other terms, it is not clear that in the diffusive regime the spatial dependence of the single particle propagator and of the local pair amplitude are governed by the same length. Moreover, we find that in a three-dimensional dirty superconductor the prefactor becomes $(k_F l)^{-1}(k_F r)^{-1}$ instead of $(k_F r)^{-2}$. Owing to the crucial importance of the prefactor, this noticeably increases the crossed effects as compared to a clean system.

This paper is organized as follows. In Section 1, using a tunneling Hamiltonian, the general result for the current across one of two neighbouring S/N interfaces is recalled. In Section 2, the calculation is performed for a dirty superconductor, within the lowest order approximation. The effect of dimensionality and geometry are discussed at the end of the paper.

Clean superconductor: tunneling interfaces

Let us consider a ballistic BCS superconductor, connected to two leads 1 and 2 (depicted in Fig. 1), with voltages V_1 and V_2 with respect to the superconductor, by tunneling contacts described by the Hamiltonian:

$$\begin{aligned}\mathcal{H}_{T1} &= \sum_{kp\sigma} T_{kp}^1 c_{k\sigma}^\dagger d_{p\sigma} + \text{h.c.}; \\ \mathcal{H}_{T2} &= \sum_{pq\sigma} T_{pq}^2 d_{p\sigma}^\dagger c_{q\sigma} + \text{h.c.}\end{aligned}\quad (1)$$

where T_{kp}^1 and T_{pq}^2 are matrix elements (hereafter assumed to be equal to T_1, T_2) between single electron states $k \in 1$, $p \in S$ and $q \in 2$.

Let us first consider single channel leads, at $T = 0$. Here we limit ourselves to lowest order results, which can also be obtained by the Keldysh technique [6–8,19] or the golden rule approximation [5,20]. Dropping the usual Andreev reflection current [20], we focus on the nonlocal contributions, e.g. the current induced in one lead by the voltage applied on the other. Using the fact that the spectral functions g_i 's ($i = 1, 2$) in the metallic leads decay on the scale of the Fermi wavelength, one obtains [5] for

the ‘‘Crossed Andreev’’ current I_{CAnd} and the ‘‘Elastic Cotunneling’’ current I_{ECot}

$$I_{CAnd} = \sum_{\sigma} \frac{4\pi^2 e}{h} |T^1 T^2|^2 \int d\omega \Xi_{12}^{And}(\omega, \sigma) \times [n_F(\omega - eV_1) - n_F(\omega + eV_2)] \quad (2)$$

$$I_{ECot} = \sum_{\sigma} \frac{4\pi^2 e}{h} |T^1 T^2|^2 \int d\omega \Xi_{12}^{Cot}(\omega, \sigma) \times [n_F(\omega - eV_1) - n_F(\omega - eV_2)] \quad (3)$$

with

$$\Xi_{12}^{And}(\omega, \sigma) = \int_1 d\vec{r}_1 \int_2 d\vec{r}_2 f_{\sigma}^r(\omega, r_{12}) \times f_{\sigma}^a(\omega, r_{21}) g_{1\sigma}(\omega) g_{2-\sigma}(-\omega) \quad (4)$$

$$\Xi_{12}^{Cot}(\omega, \sigma) = \int_1 d\vec{r}_1 \int_2 d\vec{r}_2 g_{\sigma}^r(\omega, r_{12}) \times g_{\sigma}^a(\omega, r_{21}) g_{1\sigma}(\omega) g_{2\sigma}(\omega) \quad (5)$$

where the integrals run on the contact areas, $r_{ij} = |\vec{r}_i - \vec{r}_j|$, $g_{\sigma}^r(\omega, r_{ij})$ and $f_{\sigma}^r(\omega, r_{ij})$ are respectively the time Fourier transforms of the normal $-i\langle T \{c_{i\sigma}(t), c_{j\sigma}^\dagger(0)\} \rangle$ and anomalous $i\langle T \{c_{i\sigma}(t), c_{j\sigma}(0)\} \rangle$ bare retarded Green's functions in the superconductor. Those are given in three dimensions by

$$g^r(r, \omega) = -\frac{m}{2\pi\hbar^2} \frac{1}{r} e^{-r/2\xi(\omega)} \times [\sin k_F r \frac{\omega}{\sqrt{\Delta^2 - \omega^2}} + \cos k_F r] \quad (6)$$

$$f^r(r, \omega) = -\frac{m}{2\pi\hbar^2} \frac{1}{r} e^{-r/2\xi(\omega)} \sin k_F r \frac{\Delta}{\sqrt{\Delta^2 - \omega^2}} \quad (7)$$

where $\xi_{\omega} = \xi_0 \frac{\Delta}{\sqrt{\Delta^2 - \omega^2}}$ is a generalized frequency-dependent coherence length. One can then calculate the conductances associated respectively to crossed Andreev and elastic cotunneling processes, e.g. $G_{CAnd} = dI_{CAnd}/d(V_1 + V_2)$ and $G_{ECot} = dI_{ECot}/d(V_1 - V_2)$. Up to geometrical factors, the result for interfaces of radius $a \ll \xi_0$ but much larger than the Fermi length k_F^{-1} and distant by $r \gg a$ (Fig. 1a) is [5]

$$\begin{aligned}G_{CAnd} &\sim \frac{h}{8e^2} \sum_{\sigma} G_{1\sigma} G_{2-\sigma} \frac{e^{-2r/\pi\xi_{\omega}}}{(k_F r)^2} \\ G_{ECot} &\sim \frac{h}{8e^2} \sum_{\sigma} G_{1\sigma} G_{2\sigma} \frac{e^{-2r/\pi\xi_{\omega}}}{(k_F r)^2}.\end{aligned}\quad (8)$$

Here $G_{1\sigma}$ and $G_{2-\sigma}$ are the one-electron conductances in the normal state for a given spin. As shown in references [4,5,8], both crossed conductances can be distinguished and measured as soon as leads 1 and 2 are spin-polarized. A recently proposed alternative is to use the crossed correlations of shot noise [14].

The dimensionality of the superconductor is crucial: the BCS coherence length for a clean superconductor can

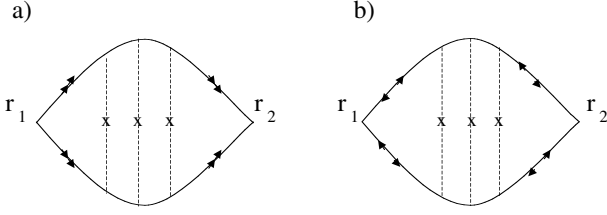


Fig. 2. The diffuson diagrams in the superconductor, showing multiple impurity scattering of quasiparticles in the normal (a) and the anomalous (b) channel. Continuous lines denote the propagators g (a) and f (b), dotted lines the impurity vertices.

be quite large, and the algebraic factor describing the ballistic propagation of quasiparticles in S is the most limiting effect in three dimensions. The constraint is weaker in two dimensions where it becomes $\sim \frac{e^{-2r/\pi\xi\omega}}{k_F r}$, and in one dimension where one finds $e^{-2r/\pi\xi\omega}$ [17].

Case of a diffusive superconductor

Disorder is always present in low dimensional superconductors (films) thus it is important to consider the diffusive limit where elastic scattering occurs with a mean-free path l . We assume no spin scattering that could be due to magnetic impurities or spin-orbit interaction. Using the golden rule or Keldysh technique, one can generalize equations (2-5) for any realization of the disorder by replacing the bare Green's functions g and f by the ones dressed by impurity scattering. On the other hand, disorder averaging implies to perform the average on the products of retarded and advanced Green's functions $g^r g^a$ and $f^r f^a$. These averages are related to the normal and the anomalous integrated diffusion probabilities [21], $\mathcal{P}(r) = \int_{-\infty}^{\infty} dt \mathcal{P}(\vec{r}_1, \vec{r}_2, t)$ for $r = r_{12}$.

$$\mathcal{P}(r_{12}) = \frac{1}{2\pi\rho_0} \overline{g_\sigma^r(r_{12}, \omega)} \overline{g_\sigma^a(r_{21}, \omega)} \quad (9)$$

$$\tilde{\mathcal{P}}(r_{12}) = \frac{1}{2\pi\rho_0} \overline{f_\sigma^r(r_{12}, \omega)} \overline{f_\sigma^a(r_{21}, \omega)} \quad (10)$$

taken at $\omega \sim \varepsilon_F$. ρ_0 is the normal state density of states in the superconductor, and $\mathcal{P}(r)$ corresponds to the diffuson in a normal metal (electron-electron channel), and $\tilde{\mathcal{P}}(r)$ is the analogue with normal propagators replaced by anomalous ones (see Fig. 2). The former describes the virtual diffusion (below the gap) of an out-of-equilibrium quasiparticle, electron or hole. The second describes the anomalous diffusion of an electron becoming a hole (with emission of a Cooper pair) or vice-versa.

The solution for the diffusons starts from the Drude-Boltzmann approximation, where the g 's and f 's are independently averaged on disorder

$$\mathcal{P}_0(r_{12}) = \frac{1}{2\pi\rho_0} \overline{g_\sigma^r(\omega, r_{12})} \overline{g_\sigma^a(\omega, r_{21})} \quad (11)$$

$$\tilde{\mathcal{P}}_0(r_{12}) = \frac{1}{2\pi\rho_0} \overline{f_\sigma^r(\omega, r_{12})} \overline{f_\sigma^a(\omega, r_{21})} \quad (12)$$

where $\overline{g^{r,a}(\omega, r)}$ and $\overline{f^{r,a}(\omega, r)}$ are obtained from the propagators $g^{r,a}(\omega, r)$ and $f^{r,a}(\omega, r)$ in the clean superconductor case, e.g. $\overline{g^{r,a}(\omega, r)} = g^{r,a}(\omega, r)e^{-r/2l}$, $\overline{f^{r,a}(\omega, r)} = f^{r,a}(\omega, r)e^{-r/2l}$ [22].

The full diffusons are obtained from the integral equation [21]

$$\begin{aligned} \mathcal{P}(\vec{r}_1, \vec{r}_2) &= 2\pi\rho_0 \\ &\times \int d\vec{r}' d\vec{r}'' \mathcal{P}_0(\vec{r}_1, \vec{r}') \Gamma(\vec{r}', \vec{r}'') \mathcal{P}_0(\vec{r}'', \vec{r}_2) \end{aligned} \quad (13)$$

$$\begin{aligned} \tilde{\mathcal{P}}(\vec{r}_1, \vec{r}_2) &= 2\pi\rho_0 \\ &\times \int d\vec{r}' d\vec{r}'' \tilde{\mathcal{P}}_0(\vec{r}_1, \vec{r}') \tilde{\Gamma}(\vec{r}', \vec{r}'') \tilde{\mathcal{P}}_0(\vec{r}'', \vec{r}_2) \end{aligned} \quad (14)$$

the vertex function Γ obeying

$$\begin{aligned} \Gamma(\vec{r}_1, \vec{r}_2) &= \gamma_e \delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{\tau_e} \\ &\times \int d\vec{r}' \Gamma(\vec{r}_1, \vec{r}') \mathcal{P}_0(\vec{r}', \vec{r}_2) \end{aligned} \quad (15)$$

$$\begin{aligned} \tilde{\Gamma}(\vec{r}_1, \vec{r}_2) &= \gamma_e \delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{\tau_e} \\ &\times \int d\vec{r}' \tilde{\Gamma}(\vec{r}_1, \vec{r}') \tilde{\mathcal{P}}_0(\vec{r}', \vec{r}_2) \end{aligned} \quad (16)$$

where $\gamma_e = (2\pi\rho_0\tau_e)^{-1}$ is the bare vertex and $\tau_e^{-1} = 2\pi\rho_0 n_i |v_i|^2 = \frac{v_i^2}{l}$ the inverse scattering time for an density n_i of impurities with potential strength v_i .

Let us consider the dirty limit $l < \xi_0$, which means that the quasiparticle encounter many collisions before decaying. Then $\mathcal{P}_0, \tilde{\mathcal{P}}_0$ decay on l while Γ a priori decays more slowly, allowing a gradient approximation

$$\begin{aligned} \Gamma(\vec{r}_1, \vec{r}_2) &\sim \gamma_e \delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{\tau_e} \Gamma(\vec{r}_1, \vec{r}_2) \langle \mathcal{P}_0(r) \rangle \\ &+ \nabla^2 \Gamma(\vec{r}_1, \vec{r}_2) \langle r^2 \mathcal{P}_0(r) \rangle \end{aligned} \quad (17)$$

$$\begin{aligned} \tilde{\Gamma}(\vec{r}_1, \vec{r}_2) &\sim \gamma_e \delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{\tau_e} \tilde{\Gamma}(\vec{r}_1, \vec{r}_2) \langle \tilde{\mathcal{P}}_0(r) \rangle \\ &+ \nabla^2 \tilde{\Gamma}(\vec{r}_1, \vec{r}_2) \langle r^2 \tilde{\mathcal{P}}_0(r) \rangle. \end{aligned} \quad (18)$$

One easily calculates

$$\langle \mathcal{P}_0(r) \rangle = \langle \tilde{\mathcal{P}}_0(r) \rangle = \tau_e \frac{\Delta^2}{\Delta^2 - \omega^2} \frac{1}{1 + \frac{l}{\xi_\omega}} \quad (19)$$

$$\langle r^2 \mathcal{P}_0(r) \rangle = \langle r^2 \tilde{\mathcal{P}}_0(r) \rangle = 2\tau_e \frac{\Delta^2}{\Delta^2 - \omega^2} \frac{l^2}{(1 + \frac{l}{\xi_\omega})^3}. \quad (20)$$

This leads to the solution, valid for $r \gg l$

$$\mathcal{P}(r) = \tilde{\mathcal{P}}(r) = \left(1 + \frac{l}{\xi_\omega}\right) \frac{\Delta^2}{\Delta^2 - \omega^2} \frac{1}{4\pi D r} e^{-r/\xi_\omega} \quad (21)$$

with

$$\xi_\omega^{-2} = (D\tau_e)^{-1} \left(1 + \frac{l}{\xi_\omega}\right)^2 \left[\frac{l}{\xi_\omega} - \frac{\omega^2}{\Delta^2} - \frac{l}{\xi_\omega} \frac{\omega^2}{\Delta^2}\right]. \quad (22)$$

To lowest order in $\frac{l}{\xi_\omega}$ and $\frac{\omega}{\Delta}$, one finds

$$\tilde{\xi}_\omega \sim \sqrt{\frac{D\tau_e\xi_\omega}{l}} \sim \sqrt{l\xi_\omega} \quad (23)$$

justifying a posteriori the above gradient approximation.

We thus find that in the dirty limit, the range of the diffusons, thus of the non-local scattering probabilities, is reduced only to the “dirty limit” coherence length, and not to the mean-free-path. As for ξ_ω , it diverges as ω approaches the superconducting gap.

We can now write the crossed conductances

$$\begin{aligned} G_{CAnd} &\sim \frac{\hbar}{8e^2} \sum_{\sigma} G_{1\sigma} G_{2-\sigma} \frac{e^{-r/\tilde{\xi}_\omega}}{\hbar\rho_0 D r} \\ G_{ECot} &\sim \frac{\hbar}{8e^2} \sum_{\sigma} G_{1\sigma} G_{2\sigma} \frac{e^{-r/\tilde{\xi}_\omega}}{\hbar\rho_0 D r}. \end{aligned} \quad (24)$$

Besides the smaller decay length, one notices the different algebraic dependence, in $1/r$ instead of $1/r^2$ for the clean limit. In more detail, the conductances vary like $\frac{1}{(k_F r)(k_F l)} e^{-r/\tilde{\xi}}$, showing that for $l < r < \tilde{\xi}$, the dirty case is more favourable than the clean one. This result holds when all the dimensions of the superconductor are larger than l . If one of them is smaller (very thin film), diffusion becomes two-dimensional and the solution of the diffusion equation leads to a dependence $\frac{1}{\sqrt{r}} e^{-r/\tilde{\xi}}$ if $r > \tilde{\xi}$ and $-\ln(\frac{r}{\tilde{\xi}})$ if $r < \tilde{\xi}$, again showing the advantage of diffusive behaviour.

One can use this result to evaluate the conductance for extended contacts 1 and 2. From equations (2–5) it is given approximately by

$$G_{CAnd,ECot} \sim \int d\vec{r}_1 d\vec{r}_2 G_{CAnd,ECot}(r_{12}). \quad (25)$$

As an example, for two linear contacts facing each other at a distance $R < \tilde{\xi}$, of length and width much larger than $\tilde{\xi}$ (Fig. 1b), one easily finds that the $1/r$ factor integrates out and $G_{CAnd,ECot} \sim e^{-R/\tilde{\xi}}$.

To summarize, we have shown that Andreev and co-tunneling processes between distinct tunneling contacts on a dirty superconductor decay on the coherence length $\tilde{\xi} = \sqrt{l\xi_0}$, and that the algebraic prefactor decreases like $1/r$ with the contact distance instead of $1/r^2$ in the clean case. For extended contacts closer than ξ the crossed conductances can be more easily observed.

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